A Review of Two-State Vector Formalism

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Abstract:

In this paper we reflect on a debate that has existed in quantum mechanics for nearly as long as the topic itself has been around. This debate concerns time-symmetry in quantum mechanics, more specifically, where quantum states can evolve forwards in time and backwards in time. This paper is a review of two-state vector formalism, the most prominent theory, and an experiment that details one such phenomena probing the heart of this theory.

Introduction

As Brian Green points out in his book, *The Fabric of the Cosmos*, the laws physics do not dictate a necessity for an asymmetric time procession as we observe it. In effect, we do not observer eggs unbreaking as often as they break, however the laws of quantum mechanics do *not* dictate this should be so but rather a symmetric time procession is allowed.

A time symmetric formulation of quantum mechanics was pioneered by Aharonov, Bergman and Lebowitz. This type of system is defined by a *two-state* vector[1]

$$\langle\phi|\,|\psi\rangle\,, \tag{1}$$

where ψ is the forward evolving quantum states defined by the results of measurements performed on the system in the past relative to the time *t* and of a backwards evolving quantum state, ϕ. This does not mean that two-state vector formalism generates different predictions than the standard single-state formulation. Rather, they both describe the same theory with the same predictions but that the single state formulation assumes that only the results of *past measurements* exist. This is in contrast to the two-state vector formulation which states that the results of *future measurements* also exist.

The obvious question then becomes why is two-state vector formalism useful if they both describe the same predictions. As it happens, there are many situations in which one may want to know how a system affected other systems in the past. Two-state vector formalism proved particularly useful in describing systems that are based upon weak measurements. One such experiment was done in Israel in 2013 by Danan, Farfurnik, and Vaidman.

History

Time symmetric interpretations of quantum mechanics are nearly as old as quantum mechanics itself, one of the first to propose such an interpretation was Walter Schottky in 1921 and others. However, the first real formalism was first developed by Satosi Watanabe in 1955, which he termed Double Interferential State-Vector Formalism.[2] Watanabe proposed that the information given by forward evolving quantum states is incomplete, and that both forward and backward evolving states are required to give a complete picture.[2]

This work would be lost for nearly a decade until Aharonov, Bergmann and Lebowitz rediscovered Watanabe's work and republished it as Two-State Vector Formalsim.[3] They claimed that prediction and retrodiction can be formally obtained by separating out initial conditions and performing a series of coherence-destroying operations. Aharonov would continue working with two-state vector formalism into the early 1990's. Vaidman and Aharnov published a series of papers around that time concerning weak measurements and the implications two-state vector formalism might play in describing the phenomena.

In more recent years, the two-state vector formalism continues to describe phenomena from experiments such as Danan, Farfurnik, and Vaidman's photon experiment using nested interferometors and super-oscillations described by Berry and Popescu in 2006.[4]

Forward and Backward Evolving Quantum States

The single forward evolving quantum state, $|\psi\rangle$, is obtained when a measurement of any observable at a time, *t*, is performed and a specific outcome is obtained. Such a state performs a unitary evolution between t_1 and t where $t_1 < t$ that is governed by the Hamiltonian, *H*.[1]

$$|\psi\rangle = U(t_{1,} t)|a\rangle \qquad (2)$$

Until future measurements yield additional information, this "now" system is the complete description of the quantum state. Meaning no measurement on the system has occurred after time, t.

We might ask if there are systems that can be described by a single *backwards* evolving quantum state $\langle\phi|$. In the present, at *t*, the future of a quantum system does not yet exist, we have made no measurement, however, it's past certainly does. This past is evolving towards the future even if we do not know it that are a result of past measurements. But how to describe this quantum system only by a single backwards evolving state? This requires two things, a measurement performed *after* time *t*, and we must erase the past of the quantum system. We are required to achieve a situation where no information from the past arrives in the future. This can be achieved if we perform a measurement on a composite system such as the one setup by Danan, Farfurnik and Vaidman. This results in a system that has equal probability to be found in any state. However this backward-evolving description can only be maintained so long as the ancillary system of the composite "fixes" the forward evolving state.

This leads to the assumption that the complete description of a system must be both $\langle\phi|$ and $|\psi\rangle$. This backwards evolving state of the coupled description is defined as the complete measurement at a time, t_2 where $t_2 > t$ for obtaining a specific but *different* outcome, represented as equation (3).[1]

$$\langle\phi| = \langle b|\, U^{\dagger}(t_2, t)\ . \tag{3}$$

When combined with the forward evolving state, equations (2) and (3) leads to the relation shown in equation (4). Thus, we have a complete description of the system.

$$\langle b|\, U^{\dagger}(t_2, t)\ U(t_{1,} t)|a\rangle = \langle \phi|\ |\psi\rangle \tag{4}$$

Weak Measurements

One of the most interesting phenomena that can be observed in the framework of Two-State Vector Formalism are related to weak measurements[1]. Weakened measurements are a standard measuring procedure with weakened coupling. This results in a measurement that provides little information, but also disturbs the system very little as well. The goal is to make a sufficiently weak that the change in the quantum state can be neglected overall. If this is achieved, it results in an outcome of a measurement that is affected by *both* states.

As an example, we can consider a simple Stern-Gerlach experiment where we measure a spin-component of a spin ½ particle. Consider a particle in the initial sate spin "up" in the $\hat{x}$ direction and it's post-measurement state to be up in the $\hat{y}$ direction. We then measure at the intermediate time, *t*, the spin component of the bisector between the two directions (σ_ξ). This results in

$$(\sigma_\xi) = \frac{\langle \uparrow_y|\sigma_\xi|\uparrow_x\rangle}{\langle \uparrow_y|\uparrow_x\rangle} = \frac{1}{\sqrt{2}}\frac{\langle \uparrow_y|(\sigma_x + \sigma_y)|\uparrow_x\rangle}{\langle \uparrow_y|\uparrow_x\rangle} = \sqrt{2} \tag{5}$$

Notice that in the standard interpretation this value is forbidden as these states can only take on values of ±1. In the usual strong measurement, where the strength of a measurement is given by Δ << 1, the probability distribution does center around ±1 state eigenvalue. However, as we weaken the measurement, where Δ is increased from say, 0.1 to 10, the observed probability distribution broadens significantly away from ±1 shown in figure (1)[1].

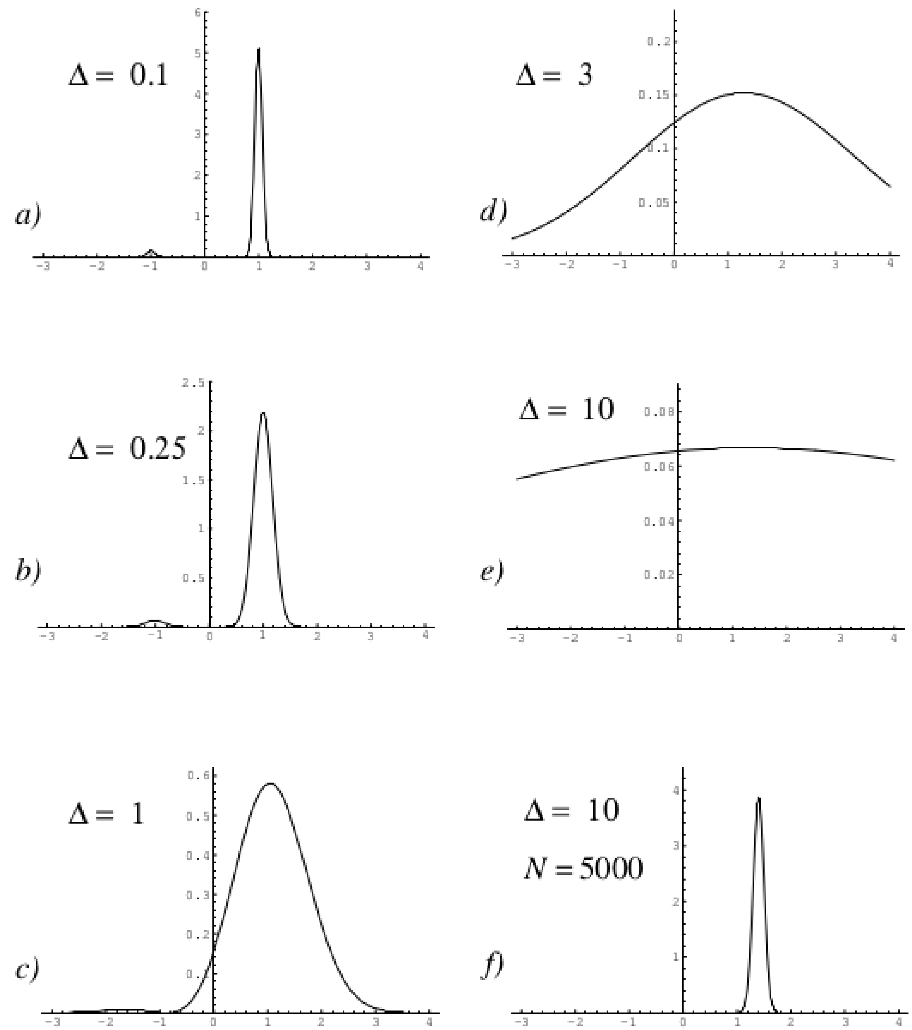

Figure 1: Measurement on pre- and post-selected ensembles: Probability distribution of the pointer variable for measurement of σ_ξ when the particle is pre-selected in the state $|\uparrow_x\rangle$ and post-selected in the state $|\uparrow_y\rangle$. The strength of the measurement is parameterized by the width of the distribution Δ. (a) $\Delta = 0.1$; (b) $\Delta = 0.25$; (c) $\Delta = 1$; (d) $\Delta = 3$; (e) $\Delta = 10$. (f) Weak measurement on the ensemble of 5000 particles; the original width of the peak, $\Delta = 10$, is reduced to $10/\sqrt{5000} \simeq 0.14$. In the strong measurements (a)-(b) the pointer is localized around the eigenvalues ± 1, while in the weak measurements (d)-(f) the peak of the distribution is located in the weak value $(\sigma_\xi)_w = \langle\uparrow_y|\sigma_\xi|\uparrow_x\rangle / \langle\uparrow_y|\uparrow_x\rangle = \sqrt{2}$. The outcomes of the weak measurement on the ensemble of 5000 pre- and post-selected particles, (f), are clearly outside the range of the eigenvalues, (-1,1).

Graph *f* in figure 1 shows that the results if the number of particles is increased in the ensemble. The probability distribution then is not observed to be so broad but centers over the value $\sqrt{2}$, not the classical ±1 eigenvalue. What we might infer from these findings is that allowed eigenvalues show themselves when the measurement is strong, IE the quantum states are forced. But we observe a very different phenomenon if we do not strongly interact with the particles and the usual eigenvalue interpretation fails, we then can only explain this through two-state vector formalism, or more to point, the standard interpretation of time-asymmetry does not provide a complete picture of quantum states.

Asking where a Photon Has Been

In 2013, Danan, Farfurnik, Bar-Ad, and Vaidman performed an experiment using nested interferometers. The team started with a Mach-Zehnder interferometer alighted in such fashion that every photon ends up in a detector. However, one modification that was made is that on the split beam they nested *another* interferometer inside the instrument. The transmitting mirror (B) in figure 1 is setup to cause complete destructive interference. The red line is the forward evolving quantum state of the photons and the green is the backwards evolving state. What the power spectrum shows in the figure is that photons were observed on mirrors A and B, inside the nested interferometer but none were observed *entering* or *leaving* the nested interferometer. They only show up where the quantum wave functions do not vanish.

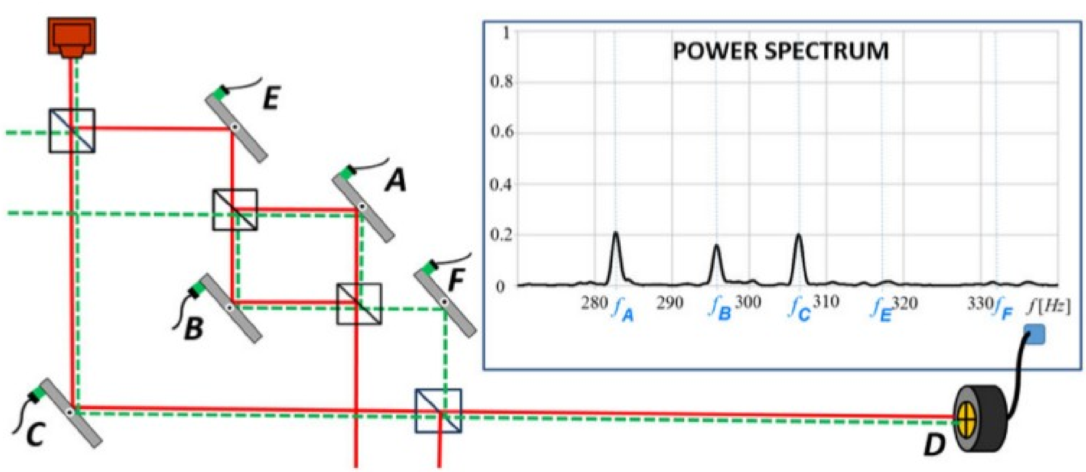

Figure 1: Experimental Setup: *Representation of the forward and backwards evolving states of the light source through the inteferometer. The power spectrum shows the detection of light on A, B and C but not E or F.*

The experiment used visible light. The premises within the framework of two-state vector formalism is that the beam has partial wave functions when the packets bounce off each mirror, altering the quantum state. When destructive interference removes either the forward evolving or backwards evolving quantum states, the whole wave packet vanishes, as seen on mirrors E and F in figure 1. Ergo, these quantum states require a time-symmetric formulation in order to exist.

Conclusion:

Almost since the dawn of quantum mechanics has time-asymmetry been debated in reference to quantum states. However as technology has increased we are able to probe some of the phenomena predicted by the time-symmetric perspective in order to find out if it exists. One such experiment's results can only be described in full by using a time-symmetric approach, like two-state vector formalism.

References

1. . Aharonov and L. Vaidman, The Two-State Vector Formalism, an Updated Review, in Time in Quantum Mechanics, Vol 1., Lect. Notes Phys. 734, 399 (2008).

2. Watanabe, Satosi,. Symmetry of Physical Laws Part III. Prediction and Retrodiction. Rev. Mod Phy, Vol. 27, #1, April 1955.

3. Y. Aharonov, D. Albert, and L. Vaidman. Phys Rev. Lett 60, 1351 1964.

4. M V Berry[1] and S Popescu[1,2] Published 16 May 2006 • 2006 IOP Publishing Ltd Journal of Physics A: Mathematical and General, Volume 39, Number 22

5. J Mehra, H Rechenberg, The Historical Development of Quantum Theory. Springer, New York, pp 303